

Combining dynamic and thermodynamic models for dynamic simulation of a beta-type Stirling engine with rhombic-drive mechanism

Chin-Hsiang Cheng*, Ying-Ju Yu

Department of Aeronautics and Astronautics, National Cheng Kung University, No.1, Ta-Shieh Road, Tainan 70101, Taiwan, ROC

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ABSTRACT

The present study is aimed at development of a dynamic model for the beta-type Stirling engines with rhombic-drive mechanism. Dynamic simulation of the engine is carried out under different operating and geometrical conditions by connecting the dynamic model to the existing thermodynamic model [23]. In the connection, the instantaneous gas forces exerted on the piston and the displacer are determined from the gas pressures in the expansion and the compression chambers with the help of the thermodynamic model. Once the gas forces are obtained, the dynamic model is used to determine the positions, velocities, and accelerations of the components of the engine at the consecutive time step, and then the gas pressures in the expansion and the compression chambers can be updated by the thermodynamic model. In this manner, transient variation in rotational speed of the engine during the hot-start period and the performance curves of the engine indicating the dependence of the power output and the thermal efficiency on the rotation speed can be predicted. In the present study, the effects of the major geometrical and operating parameters on the steady-state rotational speed, maximum power output and thermal efficiency have been evaluated thoroughly in a comprehensive parametric study.

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1. Introduction

Concentrating solar power systems with Stirling engines (CSP-Stirling) have been treated as one of the most promising renewable energy technologies and received great attention from the related researchers in the past several years [1–3]. The capacity of a typical CSP-Stirling units ranges from 3 to 25 kW. In general, the CSP-Stirling system is equipped with a solar energy absorber to receive the solar radiation and a Stirling engine to convert the received solar energy to the electrical power output. Theoretically, the performance of the CSP-Stirling system for generating electrical power is strongly dependent on the energy conversion efficiencies of the solar energy absorber and the Stirling engine. According to the earlier review of Ross [4], among all different sizes, the domestic-scale Stirling electric power generator [5] is particularly of great market potential.

Stirling engines operate over a closed, regenerative thermodynamic gas cycle, with cyclic compression and expansion of the working gas in different chambers at different temperature levels. The working gas in the Stirling engines might be air, nitrogen,

helium, or hydrogen. In terms of the arrangement of piston and displacer, the Stirling engines are classified into three types of configuration, namely alpha-, beta- and gamma-type [6]. In the past several decades, a number of experimental studies of the performance of different types of Stirling engines have been conducted [7–11]. A detailed literature review regarding the Stirling engines design was provided by Kongtragool and Wongwises [12].

On the other hand, numerical simulation based on theoretical model is also regarded as a useful tool toward understanding of the physical transport phenomena in the Stirling engines that might not be easily obtained by experiments. Schmidt [13] firstly proposed an isothermal model which has been used widely due to its mathematical simplicity. The main assumption of Schmidt's theory is that the chambers inside the engine are of the same pressure and each chamber is fixed at a constant but distinct temperature. Lately, Finkelstein [14] performed a non-isothermal analysis and evaluated the heat transfer coefficient and the temperature variation in the working space. Makhkamov and Ingham [15] divided the internal circuit of the Stirling engine into five chambers, and solved the governing ordinary differential equations for the energy and mass conservation. Mechanical losses in the construction of the engine have been determined and the computational results show that the mechanical losses for this particular design of the Stirling engine may be up to 50% of the indicated power of the engine. Recently, the

* Corresponding author. Tel.: +886 6 2757575x63627; fax: +886 6 2389940.
E-mail address: chcheng@mail.ncku.edu.tw (C.-H. Cheng).

Nomenclature			
a	acceleration (m/s^2)	R_d	offset distance from gear centers to joints between rhomboid and gears (m)
c_{load}	load coefficient	R_{t1}, R_{t2}	thermal resistances of expansion and compression chambers ($^{\circ}\text{C/kW}$)
dt	time step (s)	t	time (s)
F_1	friction force at contact surface between piston and cylinder (N)	t_p	cycle period (s)
F_2	friction force at contact surface between displacer rod and piston (N)	T_H	heat source temperature (K)
G	regenerative channel gap (m)	T_C	heat sink temperature (K)
I_3	flywheel moment of inertia (kg m^2)	W	steady-state output power (W)
l_d	length of displacer (m)	X, Y	internal forces in x- and y-direction (N)
M	moment (N m)	x, y	rectangular coordinates
m_1	mass of displacer assembly (displacer, displacer rod, and link 7–8) (kg)	y	displacement (m)
m_2	mass of piston (kg)	z_1 to z_4	geometrical constants, Eqs. (2a)–(2d)
m_3	mass of left gear (kg)	Greek Symbols	
m_4	mass of right gear (kg)	η	thermal efficiency
m_6	mass of link 6–10 and piston connecting rod (kg)	θ	angle (rad)
m_7	mass of link 5–6 (kg)	ω	instantaneous rotational speed (rad/s)
m_8	mass of link 5–7 (kg)	ω_{ave}	cycle-average rotational speed (rad/s)
m_9	mass of link 8–9 (kg)	α	angular acceleration (rad/s^2)
m_{10}	mass of link 9–10 (kg)	Φ	phase angle (degree)
m_{air}	mass of working air (kg)	τ	torque (N m)
P	pressure (kPa)	v	velocity (m/s)
Q_{in}	input heat transfer rate (W)	Subscripts	
q	crank angle (rad)	initial	initial value
r	radius (m)	d	displacer
		load	external load
		p	piston

progress in the development of numerical models for the Stirling engines has been facilitated significantly [16–21].

Rhombic-drive mechanism was proposed by the Philips company [22], and has been commonly adopted in the β -type Stirling engines. In the rhombic-drive mechanism, one rigid rod is installed connecting the piston to the top corner of a jointed rhomboid, and another rigid rod connecting the displacer to the bottom corner of the rhomboid. In addition, two symmetric gears of equal diameter are connected to the right and the left corners of the rhomboid fixed on the gears at an offset distance from gears center. The two gears are in contact and rotate in opposite directions, one of which is connected to the flywheel. When gas pressure is applied to the piston, the top corner of the rhomboid is pushed downward; the rhomboid is flattened in the direction of the piston axis; and then it pushes on the gear wheels and causes them to rotate. As the gear wheels rotate, the rhomboid experiences its change of shape by driving its top and bottom corners to move upwards and downwards, respectively. In this manner, the piston and the displacer are moved back-and-forth in the cylinder in a perfectly coaxial way. The major advantages of the rhombic-drive mechanism are that the coaxial movement of the piston and the displacer is quiet and requires no serious lubrication. Recently, Cheng and Yu [23] presented a thermodynamic model for the beta-type Stirling engine with the rhombic-drive mechanism. Periodic variation of pressures, volumes, temperatures, masses, and heat transfers in the expansion and the compression chambers are predicted by taking into account the effectiveness of the regenerative channel as well as the thermal resistances of the heating and cooling heads.

As stated earlier, a number of the thermodynamic models [13–21,23] are available for analysis of the performance of the Stirling engines; however, the existing models are limited to the thermodynamic analysis or mechanical loss in the steady-state regime and not valid for the dynamic simulation of the engines in

the start-up period. Cheng and Yu [24] firstly performed an efficient dynamic simulation model for the Stirling engines and investigated the transient variation in rotational speed of the engine during the hot-start period. Unfortunately, the model is only valid for the cam-drive mechanism and cannot be applied for the engines with rhombic-drive mechanism; therefore, a dynamic model associated with the rhombic-drive engines is still lacking. For an internal combustion engine, Giakoumis and Rakopoulos [25] provided a numerical model accounting for the transient conditions with the slider-crank-driving mechanism. The crankshaft angular momentum equilibrium was formulated by taking into account instantaneous gas, inertia and friction loads. However, the model for the internal combustion engines developed cannot be directly applied for the beta-type Stirling engines.

Under these circumstances, the present study is aimed at development of a dynamic model for the beta-type Stirling engines with the rhombic-drive mechanism. The dynamic model is built based on force and moment balances associated with all the links and components of the rhombic-drive mechanism. Dynamic simulation of the engine is carried out under different operating and geometrical conditions by combining the present dynamic model with the thermodynamic model. Effects of the major geometrical and operating parameters on the rotational speed and the performance curves of the engine are then investigated in a parametric study.

2. Dynamic model

2.1. Kinematic analysis

Fig. 1 shows a typical β -type Stirling engine with the rhombic-drive mechanism and all its parts. The 10 W engine (Model 10ST1) is designed and made in the Power Engines and Clean Energy Laboratory (PEACE Lab.), National Cheng Kung University. The

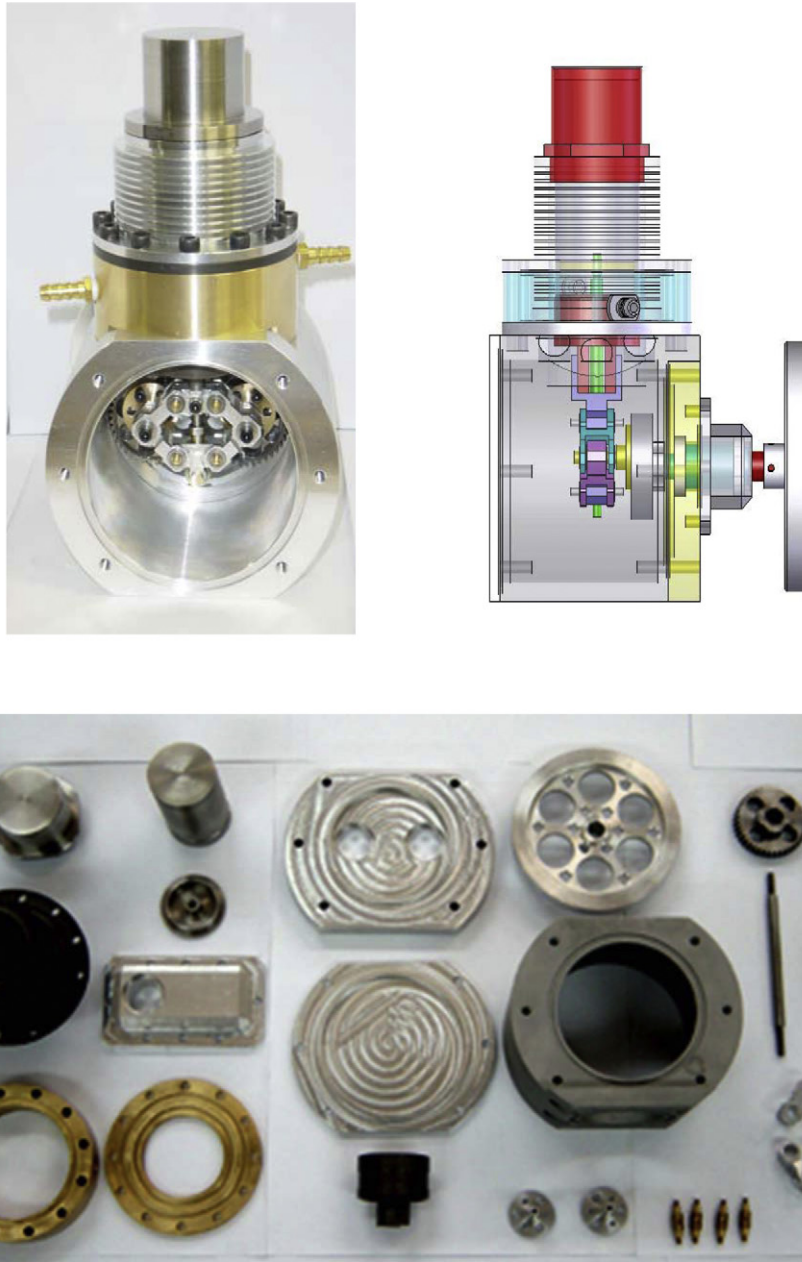


Fig. 1. A β -type Stirling engine with rhombic-drive mechanism and all its parts (Model 10ST1).

schematic of the engine is illustrated in Fig. 2, where θ represents the crank angle; θ_5 the angle of the right gear; R_d the offset distance from gear centers to the joints between the rhomboid and the gears; l_1 , l_2 , l_3 , and l_4 the lengths of the links; L_{dt} the length from the linkage l_1 to the top surface of the displacer; L_{pt} the length from the link l_4 to the top surface of the piston; L the distance between the center of the two gears; G the size of the regenerative channel; and r_1 , r_2 the radii of the displacer and the cylinder, respectively. Basically, the schematic is referred to as a planar structure [26], and the piston, displacer, and the links of the mechanism are labeled in Fig. 3. In practices, the displacer of the engine moves ahead of the piston at a fixed phase angle in order to drive the working fluid to the expansion chamber to be heated and to the compression chamber to be cooled so that the working air experiences expansion and compression periodically so as to deliver power through the piston.

Let $q = \theta$, the crank angle, and then the angles θ_5 , θ_7 , θ_8 , θ_9 and θ_{10} are expressed as:

$$\theta = q \quad (1a)$$

$$\theta_5 = \pi - q \quad (1b)$$

$$\theta_7 = \cos^{-1} (z_1 + z_2 \cos (q)) \quad (1c)$$

$$\theta_8 = \sin^{-1} (z_3 + z_4 \cos (q)) + \frac{\pi}{2} \quad (1d)$$

$$\theta_9 = \pi - \left(\sin^{-1} (z_3 + z_4 \cos (q)) + \frac{\pi}{2} \right) \quad (1e)$$

$$\theta_{10} = \pi - \left(\cos^{-1} (z_1 + z_2 \cos (q)) \right) \quad (1f)$$

where z_1 , z_2 , z_3 , and z_4 are defined as

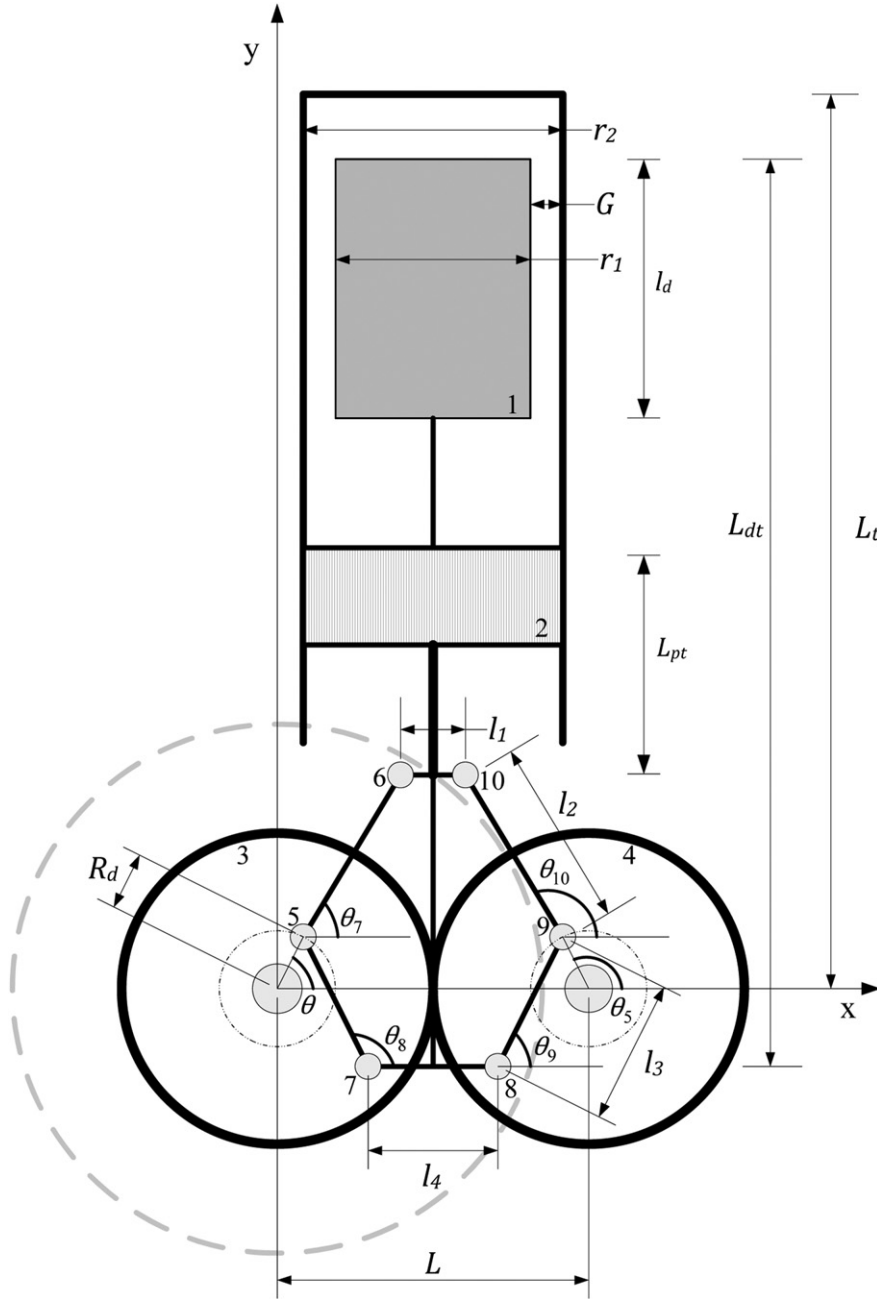


Fig. 2. Schematic of the beta-type Stirling engine with rhombic-drive mechanism.

$$z_1 = L - l_1/2l_2$$

$$(2a) \quad \omega_4 = -\omega$$

$$(3b)$$

$$z_2 = R_d/l_2$$

$$(2b)$$

$$\omega_{5-6} = \frac{z_2 \sin(q)\omega}{\sqrt{1 - (z_1 + z_2 \cos(q))^2}}$$

$$(3c)$$

$$z_3 = L - l_4/2l_3$$

$$(2c)$$

$$\omega_{5-7} = \frac{-z_4 \sin(q)\omega}{\sqrt{1 - (z_3 + z_4 \cos(q))^2}}$$

$$(3d)$$

$$z_4 = -R_d/l_3$$

$$(2d)$$

The rotating motions of the components and the links are represented by their rotation speeds, which can be determined by differentiating Eqs. (1a)–(1f). That is,

$$\omega_3 = \omega$$

$$(3a)$$

$$\omega_{9-8} = \frac{z_4 \sin(q)\omega}{\sqrt{1 - (z_3 + z_4 \cos(q))^2}}$$

$$(3e)$$

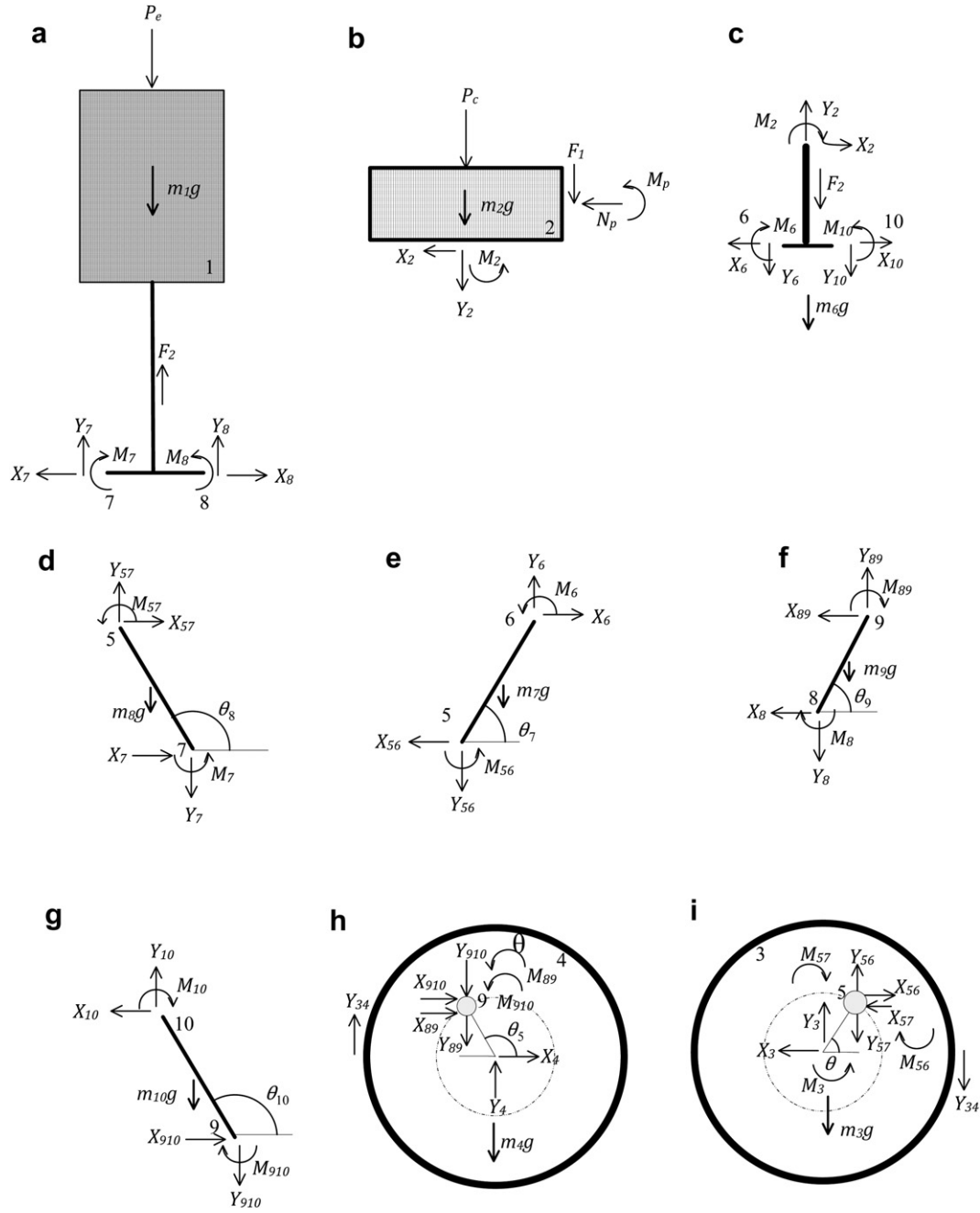


Fig. 3. Free body diagrams of the components in the rhombic-drive mechanism.

$$\omega_{9-10} = \frac{-z_2 \sin(q) \omega}{\sqrt{1 - (z_1 + z_2 \cos(q))^2}}$$

where ω is the rotational speed of the engine. If $l_2 = l_3$, then $\theta_8 = \theta_{10}$ and $\theta_1 = \theta_9$, and one has $\omega_8 = \omega_{10}$ and $\omega_7 = \omega_9$. Next, the rotational accelerations are derived by further differentiating the expressions of the rotational speeds. The results are as follows:

$$\alpha_3 = \alpha$$

(4a)

$$\alpha_4 = -\alpha$$

(4b)

$$\alpha_{5-6} = \frac{[z_2 \cos(q) \omega^2 + z_2 \sin(q) \alpha]}{\sqrt{1 - (z_1 + z_2 \cos(q))^2}} - \frac{(z_2 \sin(q) \omega)^2 (z_1 + z_2 \cos(q))}{[1 - (z_1 + z_2 \cos(q))^2]^{1.5}}$$

(4c)

$$\alpha_{5-7} = -\frac{[z_4 \cos(q) \omega^2 + z_4 \sin(q) \alpha]}{\sqrt{1 - (z_3 + z_4 \cos(q))^2}} + \frac{(z_4 \sin(q) \omega)^2 (z_3 + z_4 \cos(q))}{[1 - (z_3 + z_4 \cos(q))^2]^{1.5}}$$

(4d)

$$\alpha_{9-8} = \frac{[z_4 \cos(q)\omega^2 + z_4 \sin(q)\alpha]}{\sqrt{1 - (z_3 + z_4 \cos(q))^2}} - \frac{(z_4 \sin(q)\omega)^2 (z_3 + z_4 \cos(q))}{[1 - (z_3 + z_4 \cos(q))^2]^{1.5}} \quad (4e)$$

$$\alpha_{9-10} = -\frac{[z_2 \sin(q)\alpha + z_2 \cos(q)\omega^2]}{\sqrt{1 - (z_1 + z_2 \cos(q))^2}} + \frac{(z_2 \sin(q)\omega)^2 (z_1 + z_2 \cos(q))}{[1 - (z_1 + z_2 \cos(q))^2]^{1.5}} \quad (4f)$$

It is noted that the displacer, the piston, link 6–10, and link 7–8, experience no rotation but translation.

Displacements of the centers of mass of the displacer and the piston [$y_1(q)$ and $y_2(q)$] and the links [$y_3(q)$, $y_4(q)$, $y_{6-10}(q)$, $y_{7-8}(q)$, $y_{5-6}(q)$, $y_{5-7}(q)$, $y_{9-8}(q)$, and $y_{9-10}(q)$] are derived as:

$$y_1(q) = L_{dt} + R_d \sin(q) - l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right) \quad (5a)$$

$$y_2(q) = L_{dt} + R_d \sin(q) + l_2 \cos(\theta_7) \quad (5b)$$

$$y_3(q) = 0 \quad (5c)$$

$$y_4(q) = 0 \quad (5d)$$

$$y_{6-10}(q) = R_d \sin(q) + l_2 \sin(\theta_7) \quad (5e)$$

$$y_{7-8}(q) = R_d \sin(q) - l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right) \quad (5f)$$

$$y_{5-6}(q) = R_d \sin(q) + \frac{1}{2} l_2 \sin(\theta_7) \quad (5g)$$

$$y_{5-7}(q) = R_d \sin(q) - \frac{1}{2} l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right) \quad (5h)$$

$$y_{9-8}(q) = R_d \sin(q) - \frac{1}{2} l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right) \quad (5i)$$

$$y_{9-10}(q) = R_d \sin(q) + \frac{1}{2} l_2 \sin(\theta_7) \quad (5j)$$

The expressions for velocities of the components and the links in the y-direction are obtained by differentiating the displacement equations, Eqs. (5a)–(5j), with respect to time. The results are

$$v_1 = R_d \cos(q)\dot{q} + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8 \quad (6a)$$

$$v_2 = R_d \cos(q)\dot{q} + l_2 \sin(\theta_7)\dot{\theta}_7 \quad (6b)$$

$$v_3 = 0 \quad (6c)$$

$$v_4 = 0 \quad (6d)$$

$$v_{6-10} = R_d \cos(q)\dot{q} + l_2 \cos(\theta_7)\dot{\theta}_7 \quad (6e)$$

$$v_{7-8} = R_d \cos(q)\dot{q} + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8 \quad (6f)$$

$$v_{5-6} = R_d \cos(q)\dot{q} + \frac{1}{2} l_2 \cos(\theta_7)\dot{\theta}_7 \quad (6g)$$

$$v_{5-7} = R_d \cos(q)\dot{q} + \frac{1}{2} l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8 \quad (6h)$$

$$v_{9-8} = R_d \cos(q)\dot{q} + \frac{1}{2} l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8 \quad (6i)$$

$$v_{9-10} = R_d \cos(q)\dot{q} + \frac{1}{2} l_2 \cos(\theta_7)\dot{\theta}_7 \quad (6j)$$

The accelerations in the y-direction are obtained by differentiating the velocity equations. That is

$$a_1 = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8^2 + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\ddot{\theta}_8 \quad (7a)$$

$$a_2 = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + l_2 \sin(\theta_7)\dot{\theta}_7^2 + l_2 \cos(\theta_7)\ddot{\theta}_7 \quad (7b)$$

$$a_3 = 0 \quad (7c)$$

$$a_4 = 0 \quad (7d)$$

$$a_{6-10} = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 - l_2 \sin(\theta_7)\dot{\theta}_7^2 + l_2 \cos(\theta_7)\ddot{\theta}_7 \quad (7e)$$

$$a_{7-8} = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8^2 + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\ddot{\theta}_8 \quad (7f)$$

$$a_{5-6} = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + \frac{1}{2} \left[-l_2 \sin(\theta_7)\dot{\theta}_7^2 + l_2 \cos(\theta_7)\ddot{\theta}_7 \right] \quad (7g)$$

$$a_{5-7} = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + \frac{1}{2} \left[l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8^2 + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\ddot{\theta}_8 \right] \quad (7h)$$

$$a_{9-8} = R_d \cos(q)\ddot{q} - R_d \sin(q)\dot{q}^2 + \frac{1}{2} \left[l_3 \cos\left(\theta_8 - \frac{\pi}{2}\right)\dot{\theta}_8^2 + l_3 \sin\left(\theta_8 - \frac{\pi}{2}\right)\ddot{\theta}_8 \right] \quad (7i)$$

$$a_{9-10} = R_d \cos(q)\ddot{q} + \frac{1}{2} \left[-l_2 \sin(\theta_7)\dot{\theta}_7^2 + l_2 \cos(\theta_7)\ddot{\theta}_7 \right] \quad (7j)$$

2.2. Dynamic analysis

In order to predict the dynamic behavior of the mechanism, the free body diagrams for all the components are plotted and shown in Fig. 3. The dynamic model is built based on force and moment balances associated with all the links and components of the rhombic-drive mechanism. It is important to notice that by using the same concept of the friction cycle described in Ref. [25], the friction moments in the hinge joints, piston sliding surface, and the

contact surface of the two gears are included in the model. In addition, the effects of the weights of all components are taken into account in the model. The force and the moment balance equations associated with each free body are written as follows:

1. Displacer, displacer rod and link 7–8 [Fig. 3(a)]:

Force balance in x-direction:

$$X_8 - X_7 = m_1 a_{x1} \quad (8a)$$

Force balance in y-direction:

$$-P_e + Y_7 + Y_8 + F_2 - m_1 g = m_1 a_{y1} \quad (8b)$$

Moment balance equation:

$$(Y_8 - Y_7) \frac{l_4}{2} + M_8 - M_7 = 0 \quad (8c)$$

2. Piston [Fig. 3(b)]:

Force balance in x-direction:

$$X_2 = -N_p + m_2 a_{x2} \quad (9a)$$

Force balance in y-direction:

$$-P_c - Y_2 - \mu' N_p - m_2 g = m_2 a_{y2} \quad (9b)$$

Moment balance equation:

$$M_p = \frac{\mu' N_p r_2}{2} - M_2 \quad (9c)$$

where $-P_e + Y_7 + Y_8 + F_2 - m_1 g = m_1 a_{y1}$ is the assumed friction force exerted at the contact surface between the piston and the cylinder.

3. Link 6–10 and piston connecting rod [Fig. 3(c)]:

Force balance in x-direction:

$$X_2 + X_{10} - X_{6-10} = m_6 a_{x6-10} \quad (10a)$$

Force balance in y-direction:

$$Y_2 - Y_6 - Y_{10} - F_2 - m_6 g = m_6 a_{y6-10} \quad (10b)$$

Moment balance equation:

$$-X_2(l_{pt} - 0.5l_p) + 0.5l_1(Y_6 - Y_{10}) + M_{10} - M_6 - M_2 = 0 \quad (10c)$$

4. Link 5–7 [Fig. 3(d)]:

Force balance in x-direction:

$$X_{57} + X_7 = m_8 a_{x5-7} \quad (11a)$$

Force balance in y-direction:

$$Y_{57} - Y_7 - m_8 g = m_8 a_{y5-7} \quad (11b)$$

Moment balance equation:

$$l_3 \sin(\pi - \theta_8) X_7 - Y_7 l_3 \cos(\pi - \theta_8) - \frac{m_8 g l_3 \cos(\pi - \theta_8)}{2} - M_7 - M_{57} = I_{5-7} \alpha_{5-7} \quad (11c)$$

5. Link 5–6 [Fig. 3(e)]:

Force balance in x-direction:

$$X_{56} - X_6 = m_7 a_{x5-6} \quad (12a)$$

Force balance in y-direction:

$$Y_{56} - Y_6 - m_7 g = m_7 a_{y5-6} \quad (12b)$$

Moment balance equation:

$$-l_2 \cos(\theta_7) X_6 + Y_6 l_2 \sin(\theta_7) - \frac{m_7 g l_2 \sin(\theta_7)}{2} + M_6 + M_{56} = I_{5-6} \alpha_{5-6} \quad (12c)$$

6. Link 8–9 [Fig. 3(f)]:

Force balance in x-direction:

$$-(X_{89} + X_8) = m_9 a_{x8-9} \quad (13a)$$

Force balance in y-direction:

$$Y_{89} - Y_8 - m_9 g = m_9 a_{y8-9} \quad (13b)$$

Moment balance equation:

$$-l_3 \sin(\theta_9) X_8 + Y_8 l_3 \cos(\theta_9) + \frac{m_9 g l_3 \cos(\theta_9)}{2} + M_8 - M_{89} = I_{8-9} \alpha_{8-9} \quad (13c)$$

7. Link 9–10 [Fig. 3(g)]:

Force balance in x-direction:

$$X_{910} - X_{10} = m_{10} a_{x9-10} \quad (14a)$$

Force balance in y-direction:

$$-(Y_{910} - Y_{10}) - m_{10} g = m_{10} a_{y9-10} \quad (14b)$$

Moment balance equation:

$$l_2 \sin(\pi - \theta_{10}) X_{10} - Y_{10} l_2 \cos(\pi - \theta_{10}) + \frac{m_{10} g l_2 \cos(\pi - \theta_{10})}{2} - M_{10} - M_{910} = I_{9-10} \alpha_{9-10} \quad (14c)$$

8. Right gear [Fig. 3(h)]:

Force balance in x-direction:

$$X_{910} + X_{89} + X_4 = 0 \quad (15a)$$

Force balance in y-direction:

$$-(Y_{910} - Y_{89}) + Y_{34} + Y_4 - m_4 g = 0 \quad (15b)$$

Moment balance equation:

$$(Y_{910} + Y_{89}) R_d \cos(\pi - \theta_5) - (X_{910} + X_{89}) R_d \sin(\pi - \theta_5) - Y_{34} \frac{L}{2} + M_{910} + M_{89} = I_4 \alpha_4 \quad (15c)$$

9. Left gear [Fig. 3(i)]:

Force balance in x-direction:

$$X_{56} - X_{57} - X_3 = 0 \quad (16a)$$

Force balance in y-direction:

$$Y_{56} - Y_{57} + Y_3 - m_3g = 0 \quad (16b)$$

Moment balance equation:

$$-(X_{56} - X_{57}) R_d \sin(\theta) + (Y_{56} - Y_{57}) R_d \cos(\theta) - Y_{34} \frac{L}{2} + M_3 - M_{57} - M_{56} - \tau_{load} = I_3 \alpha_3 \quad (16c)$$

where $M_3 (= I_{11} \alpha)$ is the torque induced by the flywheel on the left gear, and α_3 is exactly equal to the engine's flywheel rotational acceleration, α , because the left gear is fixed to the flywheel. The magnitude of α_3 can be expressed by

$$\alpha_3 = (\omega_3^{n+1} - \omega_3^n) / dt \quad (17)$$

where n indicates the time step. Combining Eqs. (16c) and (17) leads to

$$\omega_3^{n+1} = \omega_3^n + \frac{dt}{I_3} [\tau_{power} - \tau_{load}] \quad (18)$$

where

$$\tau_{power} = -(X_{56} - X_{57}) R_d \sin(\theta) + (Y_{56} - Y_{57}) R_d \cos(\theta) - Y_{34} \frac{L}{2} - M_{57} - M_{56} \quad (19a)$$

$$\tau_{load} = c_{load} \omega^2 \quad (19b)$$

Note that in the above equation, τ_{power} is a combination of the effects of the gas power torque, mass inertia torque, and the friction torques (M_{57} and M_{56}) exerted on the left gear as well. On the other hand, τ_{load} denotes the resisting torque resulting from the external load. The expression of τ_{load} is of different forms in different applications. For example, when the engine is connected to a centrifugal compressor or a fan, the external load for the engine may result in a resisting torque proportional to the square of its rotational speed [25]. Therefore, in this study the load torque is assigned to be proportional to the square of the engine's rotational speed with a load coefficient c_{load} .

In this study, the instantaneous gas forces exerted on the piston and the displacer are determined from the gas pressures in the expansion and the compression chambers with the help of the thermodynamic model [23]. On the other hand, the present dynamic model is used to calculate the instantaneous rotation speed of the engine (ω) at any instant. The forces and torques exerted on the components and links of the engine at any instant are calculated by solving Eqs. (8)–(16) simultaneously as the gas pressures in the chambers are obtained from the thermodynamic model and the mass inertia, the friction force, and the external load are given. The instantaneous rotation speed of the engine ω is determined by Eq. (18), and then the value is used to determine the new positions of the piston and the displacer at the consecutive time step, and then the gas pressures in the expansion and the compression chambers can be updated by the thermodynamic model again. Detailed information regarding the thermodynamic model has been provided in Refs. [23,24]; therefore, no further information will be provided herein to save space.

Table 1 displays the geometrical and operating parameters of the baseline case. In the parametric study stated later, the variables

Table 1

Geometrical and operating variables of the baseline case.

Variable	Value
R_d (m)	0.0036
L (m)	0.042
l_1 (m)	0.018
l_2 (m)	0.018
l_3 (m)	0.018
l_4 (m)	0.018
r_1 (m)	0.020
r_2 (m)	0.0205
l_d (m)	0.07946
L_{dt} (m)	0.16374
L_{pt} (m)	0.05093
L_t (m)	0.158
m_1 (kg)	0.053
m_2 (kg)	0.028
m_3 (kg)	0.067
m_4 (kg)	0.067
m_6 (kg)	0.011
P (kPa)	101.3
T_H (K)	1000
T_c (K)	300
Working gas	air
m_{air} (kg)	3×10^{-5}
ε_{eff}	0.68
R_{r1} ($^{\circ}\text{C}/\text{kW}$)	19000.
R_{r2} ($^{\circ}\text{C}/\text{kW}$)	5000.
ϕ (degree)	90.198
Compression ratio	1.7420
Stroke (m)	0.01
Dead volume (m^3)	2.93×10^{-6}
m_7 (kg)	0.022
m_8 (kg)	0.022
m_9 (kg)	0.022
m_{10} (kg)	0.022

of the baseline case are changed to evaluate the effects of these parameters on the performance of the engine in order to find out an optimal combination of these parameters. All the studied cases are labeled in Table 2, and the geometrical and operating conditions associated with the studied cases are also given in this table.

3. Results and discussion

3.1. Baseline case analysis

Fig. 4 shows the variation in the rotational speed of the engine during the start-up period for the baseline case. In the baseline case, the heat source and heat sink temperatures are 1000 K and 300 K, respectively, and the initial charged pressure in the engine is 101.3 kPa. This figure shows the data of the instantaneous and the cycle-average rotational speeds. The engine is started from an initial rotation speed of 200 rpm, and eventually the cycle-average rotation speed reaches 915 rpm in the steady-state regime. The fluctuation of the instantaneous engine speed from the cycle-average value is not greater than 30 rpm in this case. The definition of the cycle-average rotation speed is

$$\omega_{ave}(t) = \frac{1}{t_p} \int_t^{t+t_p} \omega dt \quad (20)$$

When the steady-state regime is reached, one has $d\omega_{ave}/dt = 0$ and then a combination of Eqs. (18) and (19) leads to

$$\tau_{load} = \tau_{power} \quad (21a)$$

or

$$\omega_{ave}^2 = \tau_{power} / c_{load} \quad (21b)$$

Table 2

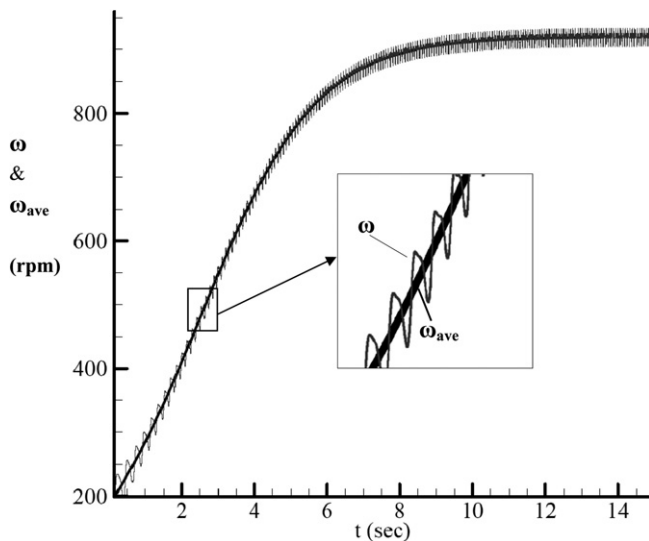
Studied cases and their steady-state rotational speed, output power, and thermal efficiency at point of maximum power output.

Case	P (kPa)	T_H (K)	R_d (m)	G (m)	I_d (m)	Φ (degree)	Compression ratio	Stroke (m)	Dead volume (c.c.)	Values at point of maximum power output		
										ω_{ave} (rpm)	W (W)	η
1-1	50.	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	912	11.4	0.4
1-2	101.3	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	789	14.5	0.435
1-3	200.	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	650	16.7	0.45
2-1	101.3	900.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	742	12.08	0.43
2-2	101.3	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	789	14.5	0.435
2-3	101.3	1100.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	826	16.9	0.44
3-1	101.3	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	789	14.5	0.435
3-2	101.3	1000.	0.0036	0.00005	0.077	90.198	1.63	0.01	6.18	748	12.21	0.36
3-3	101.3	1000.	0.0036	0.00005	0.0754	90.198	1.57	0.01	8.29	727	11.12	0.328
4-1	101.3	1000.	0.002	0.00005	0.07946	85.5	1.35	0.005	6.39	1008	7.27	0.21
4-2	101.3	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	789	14.5	0.435
4-3	101.3	1000.	0.0043	0.00005	0.07946	93.7	1.96	0.012	1.14	839	20.7	0.57
5-1	101.3	1000.	0.0036	0.00005	0.07946	90.198	1.74	0.01	2.93	789	10.5	0.435
5-2	101.3	1000.	0.0036	0.00004	0.07946	90.198	1.79	0.01	2.93	581	12.43	0.49
5-3	101.3	1000.	0.0036	0.00003	0.07946	90.198	1.84	0.01	2.93	467	13.9	0.51

Since c_{load} is a constant, it is expected that in the steady-state regime the cycle-average rotational speed is independent of the initial rotational speed. Fig. 5 reflects this expectation. Shown in this figure are the variations in the instantaneous and the cycle-average rotational speeds starting from different initial engine speeds, 150 rpm, 600 rpm, and 1200 rpm, for the baseline case. It is observed the initial rotational speed indeed has subtle influence on the variation in rotational speed in the start-up period; however, after the start-up period all different initial rotational speeds lead to the same final rotational speed in the steady-state regime.

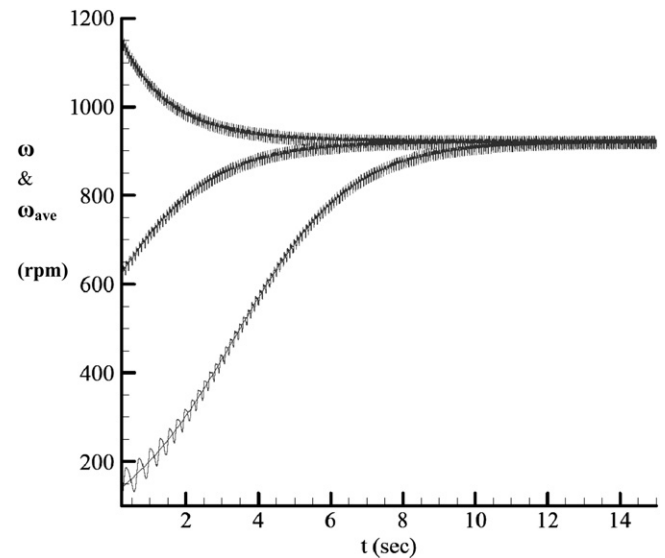
It is important to note that only an initial rotational speed within a certain range can successfully start the engine and make the engine reach the steady-state regime. It is practically reasonable that the engine is brought to a stop if an improper initial rotational speed outside the range is selected.

The flywheel moment of inertia (I_3) is regarded as one of the influential parameters that dominating the rotation of engine during the start-up period. The effects of the magnitude of the flywheel moment of inertia for the baseline case are shown in Fig. 6. In this figure, the flywheel moment of inertias is assigned to be within the range between 4×10^{-5} and 2×10^{-4} kg m². It is found that the fluctuation in the instantaneous rotation speed of the engine decreases with an increasing flywheel moment of inertia.

**Fig. 4.** Instantaneous and cycle-average engine speeds vs. time, for baseline case.

This implies that a larger moment of inertia helps reduce the fluctuation in the instantaneous rotational speed. Meanwhile, in the cases with larger moment of inertia, longer time is needed for the engine to reach the steady-state regime. However, it is observed that the steady-state cycle-average rotational speed is not changed by altering the flywheel moment of inertia. This is attributed to the fact that in the steady-state regime $d\omega_{ave}/dt = 0$ so that $I_3\alpha_3 = I_3(d\omega_{ave}/dt) = 0$. Therefore, as the steady-state regime is reached, the final cycle-average rotational speed is not affected and approaches to the same value as the moment inertia is varied within the range considered in the figure.

In Fig. 7, the variation in cycle-average rotation speed at different load torques τ_{load} for the baseline case is displayed. The load torque is changed by adjusting the value of the load coefficient c_{load} . In the cases shown in this figure, the values of c_{load} are set to be 9×10^{-8} , 3×10^{-7} , and 5×10^{-7} . In this study, the β -type Stirling engine with rhombic-drive mechanism shown in Fig. 1, Model 10ST1, is used to measure the friction loss of the engine to evaluate c_{load} . However, it is expected that the magnitude of c_{load} should be dependent on individual cases. Therefore, the test value is then extended to be 9×10^{-8} , 3×10^{-7} , or 5×10^{-7} for a parametric study.

**Fig. 5.** Instantaneous and cycle-average engine speeds vs. time, for baseline case starting from different initial speeds.

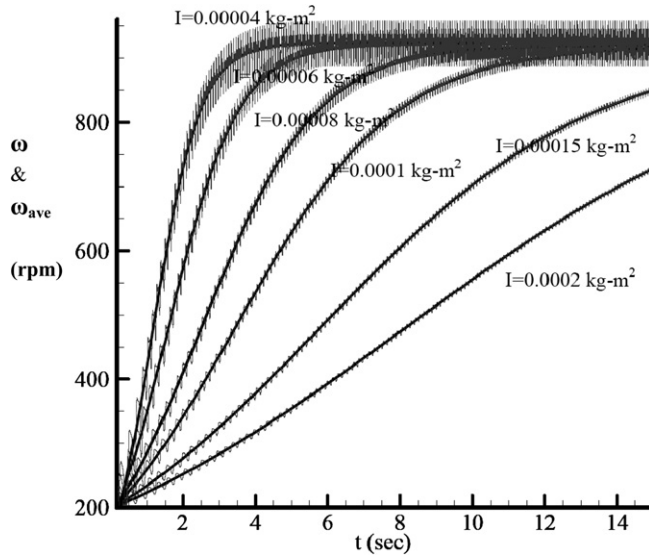


Fig. 6. Instantaneous and cycle-average engine speeds vs. time, for baseline case with different flywheel moment of inertias.

The external load torque is applied from the very beginning, and thus the engine speed is decreased due to the existence of the load. Since the load torque is assumed to be proportional to the square of its rotational speed, as the engine rotation speed increases gradually from the initial rotation speed to a higher speed, eventually the load torque is elevated to become equal to the power torque (τ_{power}) in the steady-state regime.

3.2. Parametric study

Next, the effects of geometrical and operating parameters on the performance of the engine are evaluated. The performance of the engine is represented by the power output (W) and the thermal efficiency (η) of the engine. The present model is capable of predicting the dependence of the performance of the engine on the rotation speed so that the performance curves can be plotted. The power output and the thermal efficiency are calculated after the steady-state regime is reached. The power output (W) is determined by

$$W = \tau_{load} \cdot \omega_{ave} \quad (22)$$

and the thermal efficiency (η) of the engine is expressed by

$$\eta = \frac{W}{Q_{in}} \quad (23)$$

where Q_{in} is the input heat transfer rate, which can be determined by the thermodynamic model [23]. Fig. 8 shows the performance curves associated with the baseline case. The cycle-average engine speed ranges from 100 to 2500 rpm. It is interesting to find that the power output reaches its maximum at around 900 rpm, whereas the thermal efficiency reaches its maximum at around 420 rpm. That is, the rotation speed for maximum power output is different from that for maximum thermal efficiency. Therefore, it appears that for this particular case the optimal rotation speed is within 400 and 1000 rpm, in which the magnitudes of both the power output and the thermal efficiency are relatively higher.

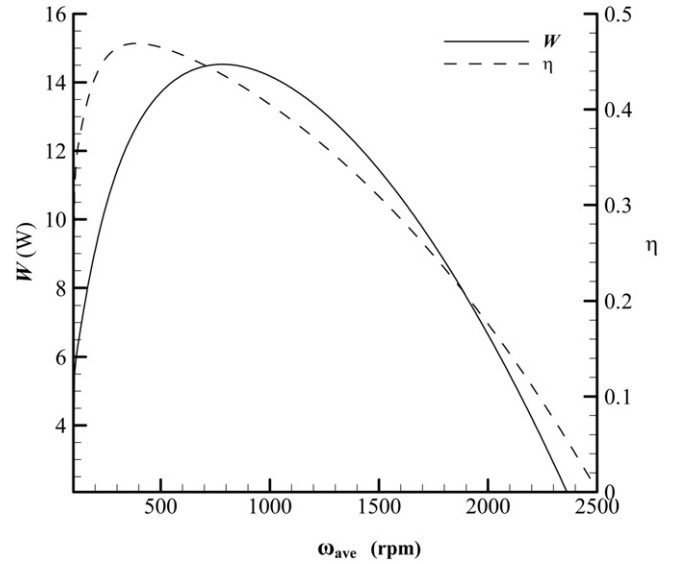


Fig. 8. Performance curves associated with the baseline case.

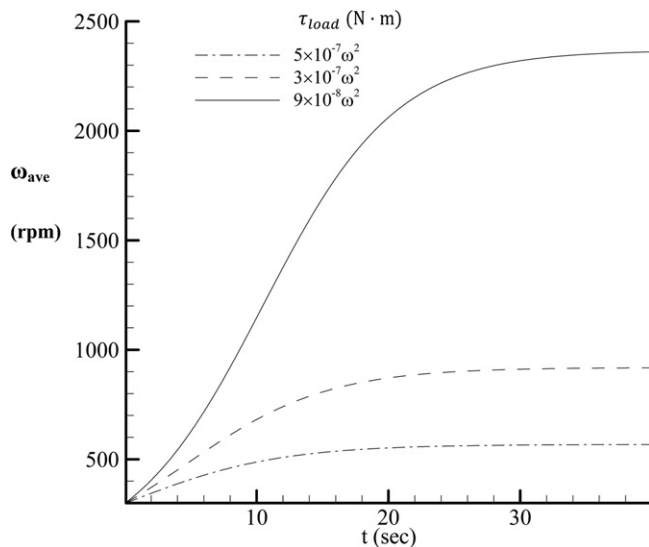


Fig. 7. Cycle-average engine speed at different load torques, for the baseline case.

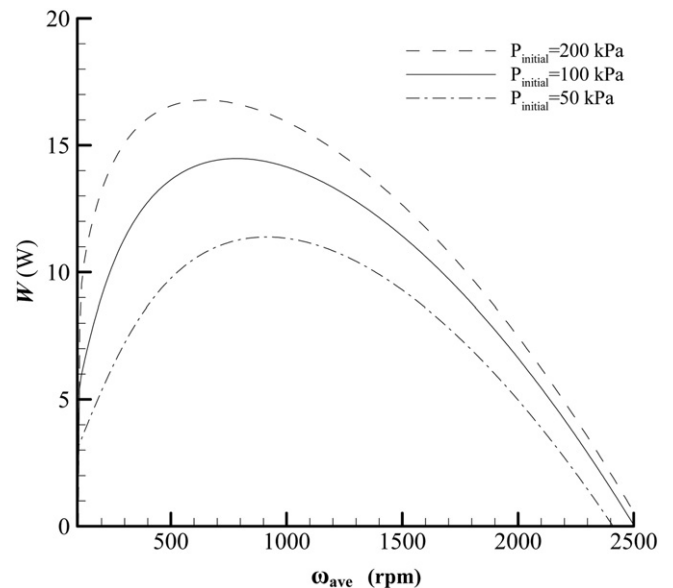


Fig. 9. Effects of initial pressure on performance curve, for Cases 1-1, 1-2, and 1-3.

Fig. 9 conveys the effects of initial charged pressure on the performance of the engine, for Cases 1-1, 1-2, and 1-3 which are listed in Table 2. In this figure, the initial pressure is set to be 50, 101.3, and 200 kPa, and the variation of the power output with the cycle-average rotation speed is plotted. It is observed that at $P_{\text{initial}} = 200$ kPa (Case 1-3), the maximum power output is 16.7 W at $\omega_{\text{ave}} = 650$ rpm. The data for the steady-state rotational speed, the output power, and the thermal efficiency for these three cases at point of maximum power output are provided in Table 2. It is noted that the larger the initial pressure, the engine speed with maximum power output is lower, and the corresponding thermal efficiency and the maximum power output are higher. In this figure, it is observed that the power output is more sensitive to a change in initial pressure in low-speed regime than in high-speed regime.

Fig. 10 shows the effects of the heat source temperature on the power output, for Cases 2-1, 2-2, and 2-3 in which the heat source temperatures are assigned to be 900, 1000, and 1100 K, respectively. For the curve for $T_H = 1100$ K, the maximum power output is 16.9 W at $\omega_{\text{ave}} = 826$ rpm. The maximum power output is significantly increased as the heat source temperature is elevated. In addition, according to the data provided in Table 2, the higher the heat source temperature, the higher is the engine's thermal efficiency. Contrary to Fig. 9, in this figure it is observed that the power output is more sensitive to a change in heat source temperature in high-speed regime than in low-speed regime. Therefore, an increase in the heat source temperature would be effective if the engine is designed to operate at higher speed.

Effects of displacer length on power output are shown in Fig. 11. It is noted that as the stroke of the piston and the phase angle are fixed, a longer displacer means a higher compression ratio and a smaller dead volume. In this figure, the length of the displacer is set to be 0.0754, 0.077, and 0.07946 m, for Cases 3-1, 3-2, and 3-3. As shown in Table 2, the corresponding compression ratios are 1.57, 1.63, and 1.74 and dead volumes 8.29, 6.18, and 2.93 cc, respectively. It is expected that the case with a higher compression and a smaller dead volume is accompanied with a higher performance. In this figure, the results presented agree with this expectation. It is found that at $l_d = 0.07946$ m, the maximum power output of 14.5 W is

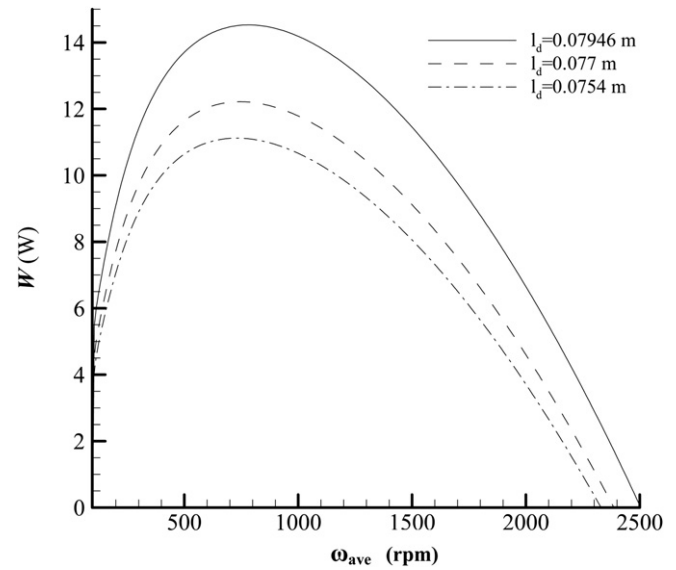


Fig. 11. Effects of displacer length on power output, for Cases 3-1, 3-2, 3-3.

attained at 789 rpm. Also indicated in Table 2, the corresponding thermal efficiency is 0.435. The maximum power output and its corresponding thermal efficiency are reduced as the length of the displacer is decreased. For the case at $l_d = 0.0754$ m, the maximum power output is only 11.12 W and the corresponding thermal efficiency is only 0.328.

The effects of the offset distance from gear centers to joints between rhomboid and gears (R_d) could be very subtle since an increase in R_d results in increases in the compression ratio and the stroke of the piston as well as a decrease in the dead volume. Therefore, one may expect that an increase of R_d might generally improve the performance. However, since the phase angle will also be altered due to a change in R_d , the influence of this parameter becomes involved. Fig. 12 conveys the influence of R_d on the performance of the engine. In this figure, the cases at $R_d = 0.002$, 0.0036, and 0.0043 m, for Cases 4-1, 4-2, and 4-3, respectively, are

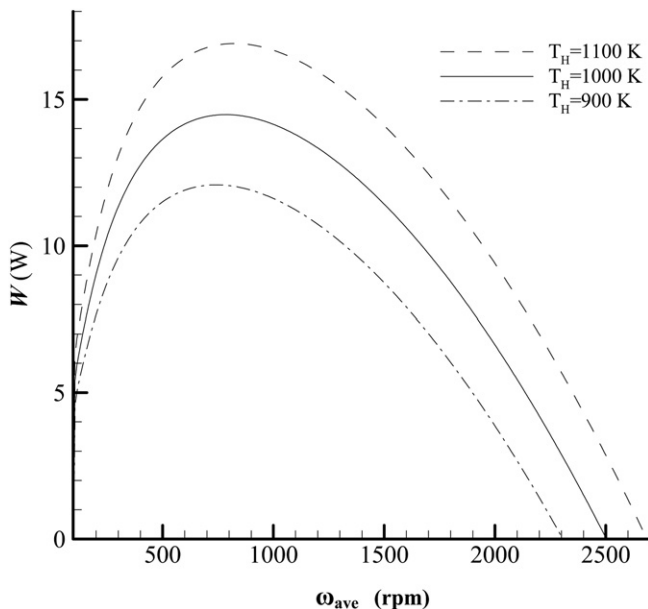


Fig. 10. Effects of heat source temperature on power output, for Cases 2-1, 2-2, and 2-3.

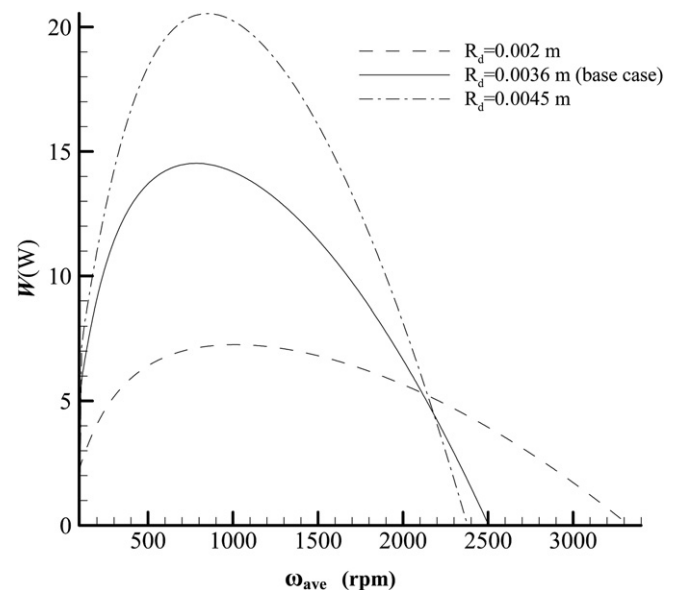


Fig. 12. Effects of R_d on power output, for Cases 4-1, 4-2, 4-3.

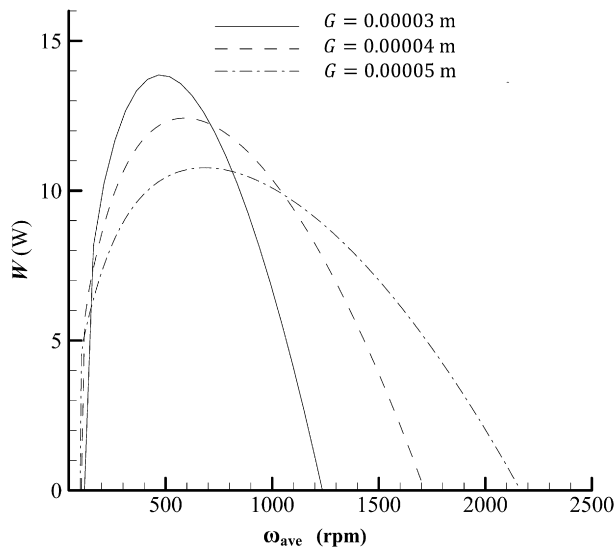


Fig. 13. Effects of regenerative gap size on power output, for Cases 5-1, 5-2, 5-3.

investigated. It is found that in most cases, the power output increases with R_d . However, a changeover is found as the rotational speed is greater than a critical value located between 2100 and 2200 rpm. In the higher rotational speed regime, larger R_d is negative to the power output. That is, in this regime the larger the value of R_d , the lower is the power output. The reason for the existence of the changeover is still not clear and open to discussion at this moment. The change in the phase angle may play an important role on the changeover of the performance, and it is worth further investigation. However, with the rhombic-drive mechanism, the phase angle is not explicit and must be derived in terms of other geometrical parameters of the mechanism. It is difficult to evaluate its sole effects by keeping other geometrical parameters fixed.

As with Cases 5-1, 5-2, and 5-3 listed in Table 2, the varied displacer radius leads to different regenerative gap sizes (G) and compression ratios of the engine while the piston stroke, the clearance volume, and the phase angle are fixed. In Fig. 13, the power outputs as functions of cycle-average rotational speed associated with the three cases are plotted. The cases are at $G = 0.00003$, 0.00004 , and 0.00005 m, which corresponding to the displacer radius of 0.00202 , 0.00201 , and 0.002 m, respectively. The maximum power output is found to be 10.5 W at 789 rpm for $G = 0.00005$ m. When the gap size is decreased, the magnitudes of the maximum power output and the corresponding thermal efficiency of the engine are increased. In the case with $G = 0.00003$ m, the optimum power output of 13.9 W is reached at a lower speed of 467 rpm. However, a decrease in the gap size is not favorable in the high-speed regime. Furthermore, a decrease in the gap size narrows the operating range of the rotational speed. For example, at $G = 0.00005$ m, the operating rotational speed ranges from approximately 100 – 2150 rpm, whereas at $G = 0.0003$ m, the engine can only operate between 100 and 1220 rpm.

4. Concluding remarks

A dynamic model for the beta-type Stirling engines with rhombic-drive mechanism has been developed and connected to the thermodynamic model [23] for dynamic simulation of the engine under different operating and geometrical conditions.

Simulation of the baseline case shows that the instantaneous rotational speed of the engine always exhibits a fluctuating

variation feature; however, the fluctuation of the instantaneous engine speed from the cycle-average value is not greater than 30 rpm. The initial rotational speed obviously affects the variation in rotational speed in the start-up period; however, all different initial rotational speeds lead to the same final rotational speed in the steady-state regime.

A larger moment of inertia helps reduce the fluctuation in the instantaneous rotational speed. Meanwhile, it is observed that the steady-state rotational speed is not changed by altering the flywheel moment of inertia. As the external load torque is applied, the engine speed is slowed down due to the existence of the load.

Results of the parametric study show that as the initial pressure or the heat source temperature is increased, the engine speed with maximum power output is lowered and the corresponding thermal efficiency and the maximum power output are elevated. The power output is more sensitive to a change in initial pressure in the low-speed regime, but it is more sensitive to a change in heat source temperature in the high-speed regime.

The maximum power output and its corresponding thermal efficiency increases with the length of the displacer or the offset distance from gear centers to joints between rhomboid and gears (R_d). However, in the higher rotational speed regime, larger R_d is negative to the power output, and thus in this regime a larger R_d may reduce the power output.

In addition, when the gap size is decreased, the magnitudes of the maximum power output and the corresponding thermal efficiency of the engine are increased; however, a decrease in the gap size narrows the operating range of the rotational speed.

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